

Homework #1

1. Consider the situation where the target density is multimodal, consisting of several widely separated peaks. (These types of distributions crop up a lot in applied economic work.) Discuss whether it is a wise strategy to do importance sampling using a source density that is unimodal and closely approximates one mode of the target density.
2. Generate 100 draws from the following distribution: $\frac{1}{\pi} \frac{1-\cos x}{x^2}$. Use both acceptance sampling and importance sampling. What are some of the features of this distribution? How does (should) that play a role in your selection of the source density?
3. Explain how you would generate draws from a standard normal distribution, using acceptance sampling with the source density being Laplace.
4. Show graphically how the Metropolis-Hastings Algorithm works.
5. Another type of sampling, called transformation sampling, was not covered in class. Transformation sampling uses known relationships between two different densities to makes draws. As an example, suppose I wanted to draw from a χ_4^2 distribution. The χ_4^2 distribution is simply the sum of four standard normal random variates squared, which are easy to draw from using a computer. First, using transformation sampling, explain how you would generate draws from a students t distribution with 7 degrees of freedom, an F distribution with 4 and 39 degrees of freedom and a beta distribution. Next, explain when it may be better to use a different method of sampling to make draws from a distribution as opposed to transformation sampling.
6. What is the benefit of drawing from a density with correlation between the draws?
7. Why should one use primes to create Halton sequences?

8. When constructing uniform random variates for higher dimensions it is typical to create points based on the corresponding elements of the Halton sequences for each dimension. for example, to create points in two dimensional space with the primes 2 and 3 the first three points would be $(1/2, 1/3)$, $(1/4, 2/3)$ and $(3/4, 1/9)$. Create 50 points (Use a computer) in two dimensional space using the primes 43 and 47.

9. Following the logic of the previous question, create Halton draws for two-dimensional space using 4 and 16 as the bases. Plot out the first 50 points. What do you notice? Why is this so?